

ADDENDUM TO “GROUP-GRADED RINGS, SMASH PRODUCTS, AND GROUP ACTIONS”

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The purpose of this addendum to [2] is twofold. First of all we give a much shorter proof of Bergman’s conjecture than our original proof in [2]; the present proof is also easier than the subsequent proofs given in [4] and [5]. Secondly, we correct two errors in [2], namely the statements of Theorem 2.10 and Corollary 2.13.

We briefly review some notation. If A is a k -algebra graded by the finite group G , let $J(A)$ denote its usual Jacobson radical and $J_G(A)$ denote its graded Jacobson radical (that is, the intersection of all annihilators of graded-irreducible A -modules). The question asked by G. Bergman in [1] was whether it was always true, for G finite, that $J_G(A) \subseteq J(A)$.

Since A is G -graded, A is an H -module algebra, for $H = (kG)^*$. Let H have as a basis the dual basis $\{p_g \mid g \in G\}$ to G ; then the grading on A is given by $A_g = p_g \cdot A$. We then form the smash product $A \# H$ (see [2, Proposition 1.4] for generators and relations).

As pointed out in [2, Lemma 2.1], M is a graded A -module $\Leftrightarrow M$ is an $A \# H$ -module; the correspondence is given simply by $M_g = p_g \cdot M$. Thus M is a graded-irreducible A -module $\Leftrightarrow M$ is an irreducible $A \# H$ -module in the usual sense.

We may now complete our proof.

THEOREM [2, THEOREM 4.4, PART (1)]. *Let A be graded by the finite group G . Then $J_G(A) \subseteq J(A)$.*

PROOF. By the above remarks about modules, it is clear that $J_G(A) = A \cap J(A \# H)$, where we are identifying A with $A \# 1$. Thus any $x \in J_G(A)$ is quasi-invertible in $A \# H$. Since $A \# H$ is free over $A \# 1$, any element of $A \# 1$ invertible in $A \# H$ is already invertible in $A \# 1$. Thus $J_G(A)$ is a quasi-regular ideal of A , so is contained in $J(A)$. \square

We remark that it was already observed by Bergman that $J(A)_G$, the graded part of $J(A)$, is contained in $J_G(A)$. Thus in fact $J_G(A) = J(A)_G$.

We now turn to Theorem 2.10 of [2]. D. S. Passman has pointed out to us an error in the proof of (2) \Rightarrow (3). For, the property of a grading being faithful assumes implicitly that $A_h \neq 0$ for every $h \in G$, whereas this is not necessary for a nondegenerate grading. The theorem can be corrected as follows:

THEOREM 2.10. *The following are equivalent:*

- (1) A_1 is prime, A is graded semiprime, and $A_g \neq 0$ for all $g \in G$.
- (2) A_1 is prime, the grading is nondegenerate, and $A_g \neq 0$ for all $g \in G$.

1980 *Mathematics Subject Classification*. Primary 16A03, 16A21; Secondary 16A24.

- (3) A_1 is prime and the grading is faithful.
- (4) $A\#(kG)^*$ is prime.

The proof in [2] is then correct, except that to prove (4) \Rightarrow (1) we must also show $A_g \neq 0$ for all $g \in G$. This follows easily since if $A_h = 0$ for some h , then $p_h(A\#(kG))^*p_1 = p_hAp_1 = A_hp_1 = 0$, a contradiction to $A\#(kG)^*$ being prime.

With this correction to 2.10, Corollary 2.11 is correct as stated, and 2.10 was not used again in the paper.

Finally we consider Corollary 2.13, in which we obtained a result of E. Dade. Without a statement as to which functor between $\text{Mod}(A_1)$ and $\text{Gr Mod}(A)$ is being considered, the corollary is false as stated (an example is given in [3]). A correct statement of Dade's result is the following:

COROLLARY 2.13. *A is strongly G -graded \Leftrightarrow the functor $M \rightarrow A \otimes_{A_1} M$ from $\text{Mod}(A_1)$ to $\text{Gr Mod}(A)$ is a category equivalence.*

Our proof of 2.13 is correct as it stands.

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